Laboratório de Otimização e Combinatória

Instituto de Computação

UNICAMP

The Online Prize-Collecting Facility Location Problem

DTC/LOCo

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Combinatorial Optimization Problems

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Problems in which an objective function needs to be minimized or maximized.

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Minimization problems in which we are interested:

- Facility Location problem,
- Prize-Collecting Facility Location problem.

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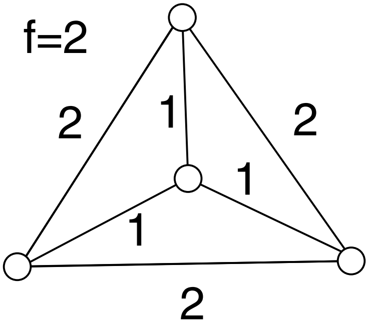
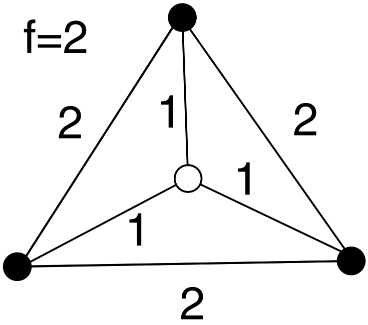
These problems are NP-hard and constant factor approximation algorithms are known for them.

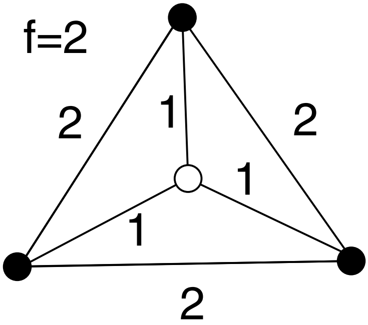
Facility Location Problem



Facility Location Problem

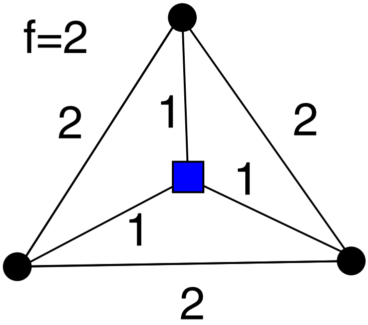


Facility Location Problem



$$\min \sum_{i \in F^a} f(i) + \sum_{j \in D} d(j, F^a)$$

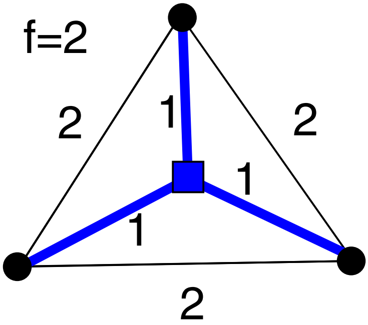
Facility Location Problem



$$\min \sum_{i \in F^a} f(i) + \sum_{j \in D} d(j, F^a)$$

Total cost $= 2$

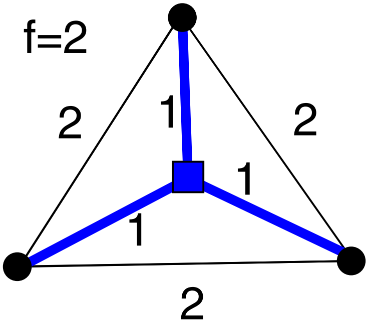
Facility Location Problem



$$\min \sum_{i \in F^a} f(i) + \sum_{j \in D} d(j, F^a)$$

Total cost $= 2 + 3$

Facility Location Problem



$$\min \sum_{i \in F^a} f(i) + \sum_{j \in D} d(j, F^a)$$

Total cost $= 2 + 3 = 5.$

Online Computation

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Each part must be served before the next one arrives.

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No decision can be changed in the future.

Competitive Analysis

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Worst case technique used to analyze online algorithms.

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An online algorithm ALG is c-competitive if:

$$\mathrm{ALG}(I) \leq c \cdot \mathrm{OPT}(I) + \kappa,$$

for every input I and some constant κ.

Online Problems

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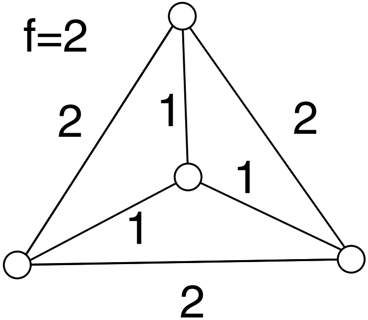
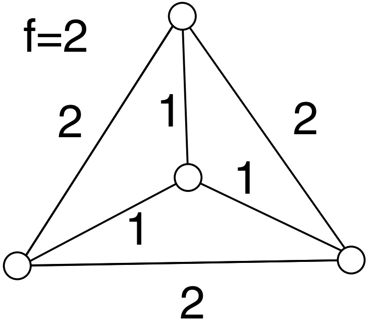
- Online Prize-Collecting Facility Location (OPFL).

Online Facility Location Problem

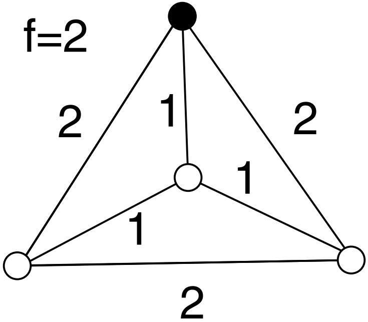


Online Facility Location Problem



$$\min \sum_{i \in F^a} f(i) + \sum_{j \in D} d(j, a(j))$$

Online Facility Location Problem



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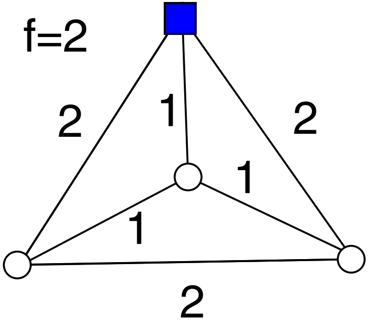
Online Facility Location Problem



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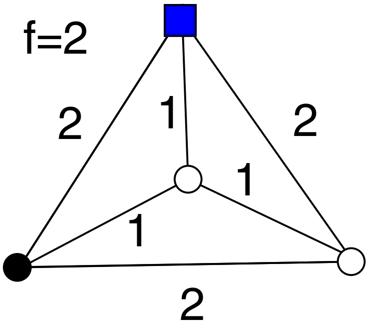
Total cost $= 2$

Online Facility Location Problem



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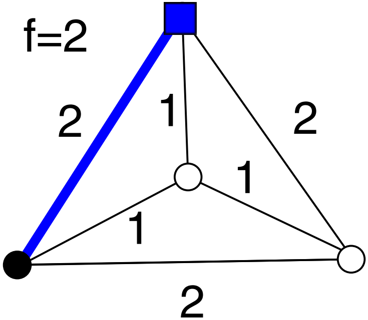
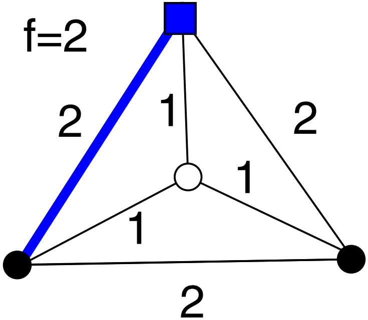
Online Facility Location Problem



$$\min \sum_{i \in F^a} f(i) + \sum_{j \in D} d(j, a(j))$$

Total cost $= 2 + 2$

Online Facility Location Problem



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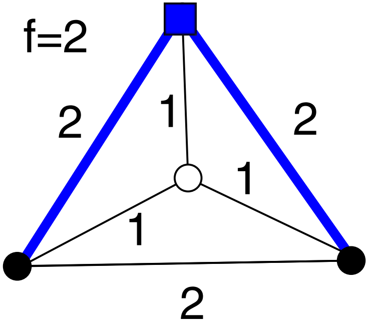
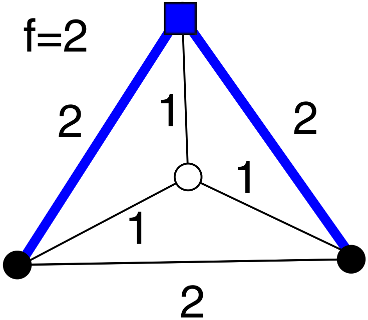


$$\min \sum_{i \in F^a} f(i) + \sum_{j \in D} d(j, a(j))$$

Total cost $= 2 + 2 + 2$

Online Facility Location Problem



$$\min \sum_{i \in F^a} f(i) + \sum_{j \in D} d(j, a(j))$$

Total cost $= 2 + 2 + 2 = 6.$

Online Facility Location Results

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The OFL has competitive ratio $\Theta\left(\frac{\log n}{\log\log n}\right)$ [Fotakis 2008].

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There are randomized and deterministic $\mathrm{O}(\log n)$-competitive algorithms known for it [Meyerson 2001, Fotakis 2007].

[Nagarajan and Williamson 2013] give a dual-fitting analysis for the algorithm by [Fotakis 2007].

Online Facility Location LP Formulation

Online Facility Location LP Formulation

Linear programming relaxation

$$
\begin{aligned}
\min \quad & \textstyle\sum_{i\in F} f(i) y_i + \sum_{j\in D}\sum_{i\in F} d(j,i) x_{ji} & \\
\text{s.t.} \quad & x_{ji} \le y_i & \text{for } j \in D \text{ and } i \in F, \\
& \textstyle\sum_{i\in F} x_{ji} \ge 1 & \text{for } j \in D, \\
& y_i \ge 0, x_{ji} \ge 0 & \text{for } j \in D \text{ and } i \in F,
\end{aligned}
$$

Online Facility Location LP Formulation

Linear programming relaxation

$$\begin{array}{lll} \min & \sum_{i\in F} f(i)y_i + \sum_{j\in D}\sum_{i\in F} d(j,i)x_{ji} & \\ \text{s.t.} & x_{ji} \le y_i & \text{for } j\in D \text{ and } i \in F, \\ & \sum_{i\in F} x_{ji} \ge 1 & \text{for } j \in D, \\ & y_i \ge 0, x_{ji} \ge 0 & \text{for } j\in D \text{ and } i\in F, \end{array}$$

and its dual

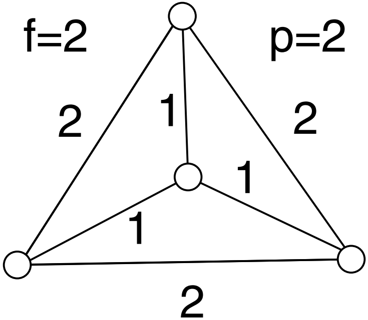
$$\begin{array}{lll} \max & \sum_{j\in D} \alpha_j & \\ \text{s.t.} & \sum_{j\in D}(\alpha_j - d(j,i))^+ \le f(i) & \text{for } i \in F, \\ & \alpha_j \ge 0 & \text{for } j \in D. \end{array}$$

Online Facility Location Algorithm

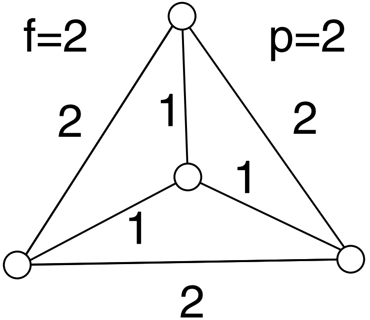
Algorithm 1: OFL Algorithm.

Input: (G, d, f, F)
$F^a \leftarrow \emptyset$; $D \leftarrow \emptyset$;
while *a new client j' arrives* **do**
    increase $\alpha_{j'}$ until one of the following happens:
    (a) $\alpha_{j'} = d(j', i)$ for some $i \in F^a$; /* connect only */
    (b) $f(i) = (\alpha_{j'} - d(j', i)) + \sum_{j \in D}(d(j, F^a) - d(j, i))^+$ for some $i \in F \setminus F^a$; /* open and connect */
    $F^a \leftarrow F^a \cup \{i\}$; $D \leftarrow D \cup \{j'\}$; $a(j') \leftarrow i$;
end
return (F^a, a);

Online Prize-Collecting Facility Location

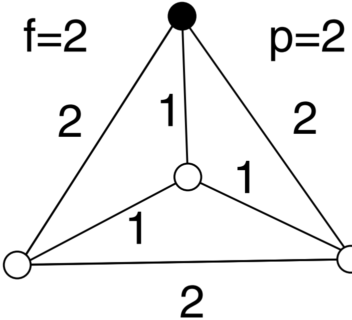


Online Prize-Collecting Facility Location



$$\min \sum_{i \in F^a} f(i) + \sum_{j \in D^c} d(j, a(j)) + \sum_{j \in D^p} p(j)$$

Online Prize-Collecting Facility Location



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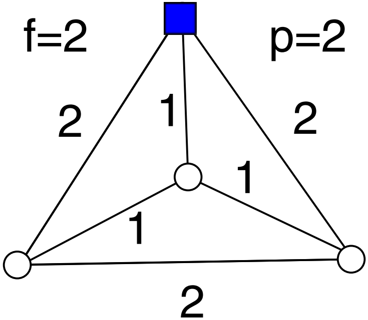
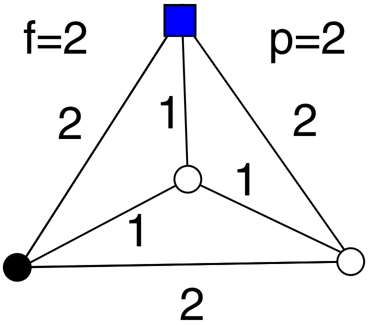


$$\min \sum_{i \in F^a} f(i) + \sum_{j \in D^c} d(j, a(j)) + \sum_{j \in D^p} p(j)$$

Total cost $= 2$

Online Prize-Collecting Facility Location



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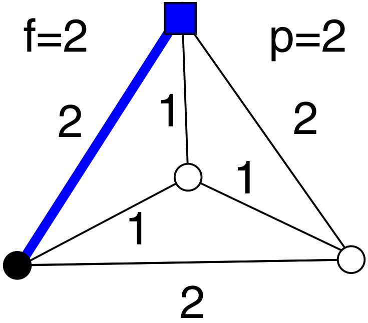
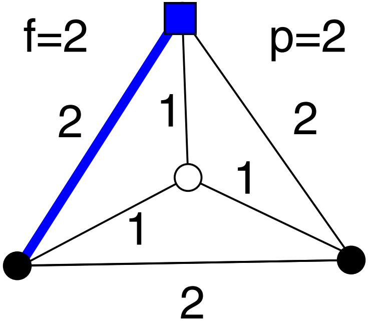


$$\min \sum_{i \in F^a} f(i) + \sum_{j \in D^c} d(j, a(j)) + \sum_{j \in D^p} p(j)$$

Total cost $= 2 + 2$

Online Prize-Collecting Facility Location



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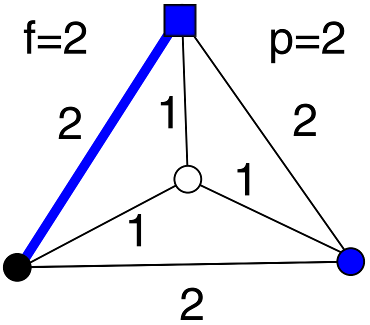

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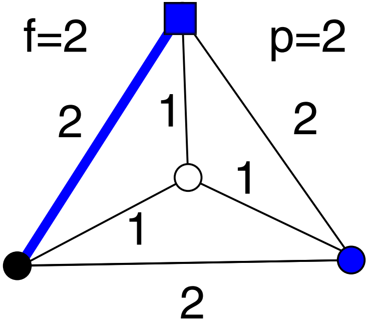

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Total cost $= 2 + 2 + 2 = 6$.

OPFL Results

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Our contribution: we proposed the problem and showed a primal-dual $(6 \log n)$-competitive algorithm for it, by extending the algorithm from [Fotakis 2007, Nagarajan and Williamson 2013].

OPFL Results

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Since the OPFL is a generalization of the OFL, the lower bound of $\Omega\left(\frac{\log n}{\log \log n}\right)$ applies to it.

OPFL LP Formulation

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Linear programming relaxation

$$
\begin{array}{lll}
\min & \sum_{i\in F} f(i)y_i + \sum_{j\in D}\sum_{i\in F} d(j,i)x_{ji} + \sum_{j\in D} p(j)z_j & \\
\text{s.t.} & x_{ji} \le y_i & \text{for } j \in D \text{ and } i \in F, \\
& \sum_{i\in F} x_{ji} + z_j \ge 1 & \text{for } j \in D, \\
& y_i \ge 0, x_{ji} \ge 0, z_j \ge 0 & \text{for } j \in D \text{ and } i \in F,
\end{array}
$$

OPFL LP Formulation

Linear programming relaxation

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$$\begin{array}{lll} \max & \sum_{j\in D} \alpha_j & \\ \text{s.t.} & \sum_{j\in D}(\alpha_j - d(j,i))^+ \le f(i) & \text{for } i\in F, \\ & \alpha_j \le p(j) & \text{for } j\in D, \\ & \alpha_j \ge 0 & \text{for } j\in D. \end{array}$$

OPFL Algorithm

Algorithm 2: OPFL Algorithm.

Input: (G, d, f, p, F)
$D \leftarrow \emptyset$; $F^a \leftarrow \emptyset$;
while *a new client j' arrives* **do**
  increase $\alpha_{j'}$ until one of the following happens:
  (a) $\alpha_{j'} = d(j', i)$ for some $i \in F^a$; /* connect only */
  (b) $f(i) = (\alpha_{j'} - d(j', i)) + \sum_{j \in D}(\min\{d(j, F^a), p(j)\} - d(j, i))^+$ for some $i \in F \setminus F^a$; /* open and connect */
  (c) $\alpha_{j'} = p(j')$; /* pay the penalty */
  (in this case i is choose to be null, i.e., $\{i\} = \emptyset$)
  $F^a \leftarrow F^a \cup \{i\}$; $D \leftarrow D \cup \{j'\}$; $a(j') \leftarrow i$;
end
return (F^a, a);

Analysis

Towards the $O(\log n)$ competitive ratio

1. Resulting assignment is feasible.

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1. Resulting assignment is feasible.
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3. $\left\{\frac{\alpha_j}{3H_n}\right\}_j$ is feasible to the dual problem, so $\sum_j \frac{\alpha_j}{3H_n} \leq \mathrm{OPT}$.

Analysis

Towards the $O(\log n)$ competitive ratio

1. Resulting assignment is feasible. ✓
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Analysis

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Analysis

Towards $O(\log n)$ competitive ratio

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D^c := {connected clients} D^p := {penalized clients}

Analysis

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D^c := {connected clients} D^p := {penalized clients}

(i) $\sum_{j \in D^c} \left(\dfrac{\alpha_j}{3H_n} - d(j,i)\right)^+ \le \dfrac{f_i}{2}$ for each $i \in F$.

Analysis

3. $\left\{\dfrac{\alpha_j}{3H_n}\right\}_j$ is feasible to the dual problem, so $\sum_j \dfrac{\alpha_j}{3H_n} \leq \mathrm{OPT}$.

$D^c := \{\text{connected clients}\}$ $D^p := \{\text{penalized clients}\}$

(i) $\sum_{j \in D^c} \left(\dfrac{\alpha_j}{3H_n} - d(j,i)\right)^+ \leq \dfrac{f_i}{2}$ for each $i \in F$.

(ii) $\sum_{j \in D^p} \left(\dfrac{\alpha_j}{3H_n} - d(j,i)\right)^+ \leq \dfrac{f_i}{2}$ for each $i \in F$.

Analysis: (i) $\sum_{j\in D^c}\left(\frac{\alpha_j}{3H_n} - d(j,i)\right)^+ \leq \frac{f_i}{2}$ for each $i \in F$

Techniques in [NW, 2013]

For each $i \in F$,

$$f(i) \geq (\alpha_{[k]} - d(j_{[k]}, i)) + \sum_{j \in D^c_{[k-1]}} \left(\min\{d(j, F^a_{[k]}), p(j)\} - d(j,i)\right)^+$$

(j connected $\Rightarrow$ $d(j, F^a_{[k]})$ is smaller)

$$= (\alpha_{[k]} - d(j_{[k]}, i)) + \sum_{j \in D^c_{[k-1]}} \left(d(j, F^a_{[k]}) - d(j,i)\right)^+$$

(triangle inequality)

$$\geq (\alpha_{[k]} - d(j_{[k]}, i)) + \sum_{j \in D^c_{[k-1]}} \left(\alpha_{[k]} - d(j_{[k]}, i) - 2d(j,i)\right)^+$$

$$\Rightarrow f(i) \geq (1 + \underbrace{(k-1)}_{=|D^c_{[k-1]}|}) \cdot (\alpha_{[k]} - d(j_{[k]}, i)) - 2\sum_{j \in D^c_{[k-1]}} d(j,i)$$

Analysis: (i) $\sum_{j \in D^c} \left(\frac{\alpha_j}{3H_n} - d(j,i)\right)^+ \leq \frac{f_i}{2}$ for each $i \in F$

Techniques in [NW, 2013]

For each $i \in F$,

$$f(i) \geq k \cdot (\alpha_{[k]} - d(j_{[k]}, i)) - 2 \sum_{j \in D^c_{[k-1]}} d(j,i)$$

$$\Rightarrow \quad \frac{f(i)}{k} \geq (\alpha_{[k]} - d(j_{[k]}, i)) - 2 \frac{\sum_{j \in D^c_{[k-1]}} d(j,i)}{k}$$

$$\Rightarrow \quad H_{|D^c|} \cdot f(i) \geq \sum_{k=1}^{|D^c|} (\alpha_{[k]} - d(j_{[k]}, i)) - 2 \sum_{k=1}^{|D^c|} \frac{1}{k} \cdot \sum_{j \in D^c_{[k-1]}} d(j,i)$$

$$\Rightarrow \quad H_{|D^c|} \cdot f(i) \geq \sum_{k=1}^{|D^c|} (\alpha_{[k]} - d(j_{[k]}, i)) - (2H_{|D^c|} - 1) \cdot \sum_{j \in D^c} d(j,i)$$

$$\Rightarrow \quad H_{|D^c|} \cdot f(i) \geq \sum_{k=1}^{|D^c|} (\alpha_{[k]} - 2H_{|D^c|} d(j_{[k]}, i))$$

Analysis: (i) $\sum_{j \in D^c} \left(\frac{\alpha_j}{3H_n} - d(j,i) \right)^+ \leq \frac{f_i}{2}$ for each $i \in F$ ✓

Techniques in [NW, 2013]

For each $i \in F$,

$$f(i) \geq k \cdot (\alpha_{[k]} - d(j_{[k]}, i)) - 2 \sum_{j \in D^c_{[k-1]}} d(j,i)$$

$$\Rightarrow \quad \frac{f(i)}{k} \geq (\alpha_{[k]} - d(j_{[k]}, i)) - 2 \frac{\sum_{j \in D^c_{[k-1]}} d(j,i)}{k}$$

$$\Rightarrow \quad H_{|D^c|} \cdot f(i) \geq \sum_{k=1}^{|D^c|} (\alpha_{[k]} - d(j_{[k]}, i)) - 2 \sum_{k=1}^{|D^c|} \frac{1}{k} \cdot \sum_{j \in D^c_{[k-1]}} d(j,i)$$

$$\Rightarrow \quad H_{|D^c|} \cdot f(i) \geq \sum_{k=1}^{|D^c|} (\alpha_{[k]} - d(j_{[k]}, i)) - (2H_{|D^c|} - 1) \cdot \sum_{j \in D^c} d(j,i)$$

$$\Rightarrow \quad H_{|D^c|} \cdot f(i) \geq \sum_{k=1}^{|D^c|} (\alpha_{[k]} - 2H_{|D^c|} d(j_{[k]}, i))$$

Analysis: (ii) $\sum_{j \in D^p} \left(\frac{\alpha_j}{3H_n} - d(j,i)\right)^+ \leq \frac{f_i}{2}$ for each $i \in F$

Same trick for D^p?

For each $i \in F$,

$$f(i) \geq (\alpha_{[k]} - d(j_{[k]}, i)) + \Big(\sum_{j \in D^p_{[k-1]}} \min\{d(j, F^a_{[k]}), \underbrace{p(j)}_{=\alpha_j}\} - d(j,i)\Big)^+$$

(j not connected $\not\Rightarrow d(j, F^a_{[k]})$ is smaller) $\quad \bar{D}^p_{[k-1]} := \{j \in D^p_{[k-1]} : \alpha_j \geq d(j, F^a_{[k]})\}$

$$\geq (\alpha_{[k]} - d(j_{[k]}, i)) + \sum_{j \in \bar{D}^p_{[k-1]}} \left(d(j, F^a_{[k]}) - d(j,i)\right)^+$$

(triangle inequality)

$$\geq (\alpha_{[k]} - d(j_{[k]}, i) + \sum_{j \in \bar{D}^p_{[k-1]}} (\alpha_{[k]} - d(j_{[k]}, i) - 2d(j,i))$$

$$\Rightarrow f(i) \geq (1 + \underbrace{|\bar{D}^p_{[k-1]}|}_{\text{could } <(k-1)}) \cdot (\alpha_{[k]} - d(j_{[k]}, i)) - 2\sum_{j \in D^c_{[k-1]}} d(j,i)$$

Analysis: (ii) $\sum_{j \in D^p} \left(\frac{\alpha_j}{3H_n} - d(j,i)\right)^+ \leq \frac{f_i}{2}$ for each $i \in F$

How to fix?

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How to fix?

Goal: harmonic series as coefficients, e.g.

$$f(i) \geq k \cdot (\alpha_k - d(j_k, i)) \qquad (*)$$

for some ordering $\{j_k\}$ over D^p.

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Analysis

Towards $O(\log n)$ competitive ratio

1. Resulting assignment is feasible. ✓
2. Total cost of the assignment is bounded by $2 \cdot \sum_j \alpha_j$. ✓
3. $\left\{ \frac{\alpha_j}{3H_n} \right\}_j$ is feasible to the dual problem, so $\sum_j \frac{\alpha_j}{3H_n} \leq \mathrm{OPT}$. ✓

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Conclusion

Our algorithm has $(6 \log n)$-competitive ratio.

Future Research:

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Other variants of Facility Location, like:

- Online Robust Facility Location,
- Online Multicommodity Facility Location,
- Online Prize-Collecting Facility Leasing.

Acknowledgements

Thank you!

Questions?

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